Secondary link importance:
Links as rerouting alternatives during road network disruptions

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Abstract. We consider the importance of road links as rerouting alternatives when some other link in the network is disrupted (due to events such as floods, landslides, car accidents or rockfall). We refer to this concept as secondary importance and introduce two measures based on traffic flows and travel delays, respectively. In the flow-based measure we consider the traffic flow that is redirected to the studied link when other links are disabled. In the delay-based measure, we also consider how much worse the next-best alternative would be if the studied link itself would fail. These measures should be useful as quantitative decision support in the allocation of resources for investments and maintenance as well as for setting up pre-emptive rerouting plans. The measures are applied in a case study of northern Sweden and the general characteristics that determine which links are secondary important are identified.

Key words: Road network, vulnerability, importance, rerouting, alternative, disruption, secondary

1 Introduction

A link in the road network can be considered important if many people typically make use of the link when they travel. In that case, many people rely on the service provided by the link and would be affected by any change in that service, such as an improvement of the travel safety. The level of use of a link corresponds to the traffic *flow* across the link (e.g. vehicles or persons per unit time); hence, we will refer to this way of measuring importance as *flow-based importance.*

Although flow-based importance measures how many travellers use the link, it does not capture how much worse off these travellers would be if the link was not available. Another way to define the importance of a link is hence to consider the impact if the link would be disrupted and impossible to utilize. This is the predominant point of departure in network vulnerability and robustness analysis ([4, 9, 8, 1, 6, 7, 5]). In this paper we take the same approach as [2, 3] and use delays, including both increases in travel time and trip postponements, as an indicator of the disruptions impacts. We will refer to this notion of importance as *delay-based importance*.

In this paper, in contrast to previous studies, we consider the importance of links as rerouting alternatives when some other link in the road network is disrupted (due to any kind of event, such as floods, landslides, car accidents or rockfall). In this setting, a road link can be considered important if many travellers are able to utilize the link as part of an alternative route when their usual route is disrupted. Furthermore, the link can be considered particularly important if the next best alternative without the link would be considerably poorer. In other words, the link provides important redundancy in the network. We will refer to link importance in this sense as *secondary importance*, in order to distinguish it from the traditional sense which we will refer to as *primary importance*.

In parallel with the flow-based and delay-based measures of primary importance, we define two corresponding measures of secondary importance. In the analysis we assume that all travellers act independently in order to minimize their own travel times. However, the measures could also be used to study the effects of imposing different authoritative rerouting schemes during a failure, such as requiring all heavy vehicles to use certain roads. In an emergency situation it could be valuable to divert traffic to other routes than the shortest ones in order to reduce the probabilities and consequences of additional failures or capacity excesses.

In Section 2 and 3 we define measures of primary and secondary importance formally. In Section 4 we introduce a case study of a part of northern Sweden in which the importance measures are applied. Results from the study are presented in Section 5, followed by some concluding remarks in Section 6.

2 Primary link importance

We first formalize two measures of primary importance based on link flow and travel delays caused by a closure. These measures have been studied extensively elsewhere [4, 2], and are used here mainly as a point of departure for our new measures of secondary importance defined in the following section. They will also be used as baselines for comparisons in the subsequent case study.

2.1 Flow-based primary importance

A simple measure of the importance of a link is the flow of vehicles or persons per unit time that traverse the link during the period of interest (e.g., the average flow during the morning peak or the entire day). Letting f_k denote the flow on link k, the flow-based primary importance of k is thus

$$I_1^{\text{flow}}(k) = f_k. \tag{1}$$

Flow is a relevant importance measure in the sense that it expresses the number of users that rely on the link for their transportation. If link k is a *cut link*, i.e., if k is the only connection between two components of the road network, then f_k is the number of users per unit time that are unable to reach their destination if k were to be closed.

2.2 Delay-based primary importance

If link k is not a cut link, so that the users can redirect their trips to alternative routes, then f_k does not fully capture the consequences if k were to be closed. An alternative approach is to measure the change in the performance of the transport system that a closure of k would cause. We base our importance measures in this paper on delays and assume that travellers' destination and mode choices are not affected by link closures.[1] The primary delay-based importance of link k can then be written as

$$\mathrm{I}_1^{\mathrm{delay}}(k) = \Delta T^k, \tag{2}$$

where ΔT^k is the total delay across all origin/destination (OD) relations. Here and in the following, superscripts refer to links being closed, while subscripts can refer to origins/destinations and links in general.

Under the assumption that only travellers normally using a link incur delays when it is closed, it can be shown formally that the delay-based primary importance of a link will be large if either the link flow or the average user delay (in essence, the absence of good alternative routes) is large, and particularly if both are large.

3 Secondary link importance

3.1 An illustrative example

To illustrate the ideas behind the new measures, consider the small example road network in Figure 1. Assuming that all links allow travel at the same speed, it is reasonable that most people travelling between A and B normally use the route (a, d). If link a is closed, then (c, f, e) is the shortest route between A and B. Hence, the flow between A and B is redirected to the links c, f and e, which serve as alternatives to a. The situation is analogous if link d is closed. Both links a and d thus contribute to the secondary importance of c, f and e. In this example, no other origin/destination pairs than (A, B) appear to be affected by closures of a or d.

Trips between A and C, meanwhile, should normally use the direct link c. When c is closed, the best alternative route is (a, b), and the flow between A and C is redirected to these links. Link c thus contributes to the secondary importance of a and b. Users travelling between B and C, finally, should normally use the route (e, f). If any of these two links is closed, the best alternative route is (d, b), and the flow will be redirected to these links.

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[1] Empirical evidence suggest that this is often a reasonable approximation, see e.g. [10].

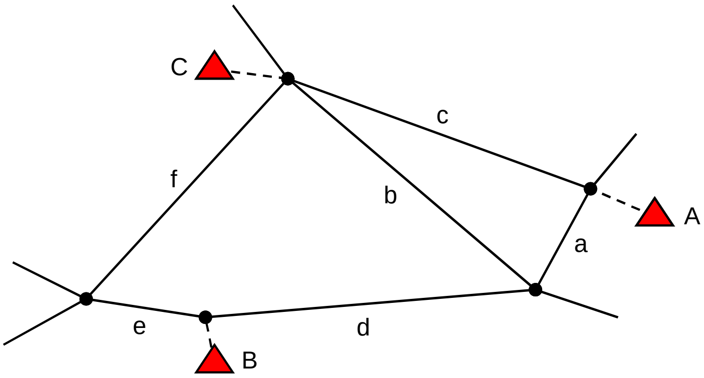

Fig. 1. Example network.

All in all, link b constitutes a rerouting alternative for links c, e and f. That is, even though link b is normally used by very few travellers in this network, it is heavily relied on as a backup link when other links are disrupted. It is therefore of low primary importance but high secondary importance. Link c, on the other hand, is not used as a rerouting alternative by any travellers, even though it is normally used by many. It is therefore of high primary importance but low secondary importance. The remaining links (a, d, e and f) are used *both* under normal conditions and as rerouting alternatives. Hence, they are of high primary as well as secondary importance.

3.2 Flow-based secondary importance

In our first measure of secondary importance, we consider the magnitude of flow that is redirected to the considered link when other links are closed. The flow-based secondary importance of k with respect to link ℓ is defined as

$$I_{2,\ell}^{\text{flow}}(k) = f_{k+}^{\ell}, \tag{3}$$

where f_{k+}^{ℓ} denotes the flow that is redirected to k when ℓ is closed. This is, in general, not equivalent to the *change* in flow on k due to the closure, because flow can also be diverted away from k. By only considering positive contributions in flow, we ensure that the flow-based secondary importance measure will always be positive. It also provides a basis for a natural definition of the delay-based importance measure introduced below.

3.3 Secondary delay-based importance

Although the flow-based secondary importance measure captures how many users take advantage of link k as a rerouting alternative for the closed link ℓ, it does not consider how much poorer the next best alternative would have been if link k had *not* been available. In the extreme, k may be part of the *only* alternative route for ℓ. For example, consider link d in Figure 1. If link e is closed, then all trips to and from B must use link d. If d too is closed, these trips cannot be completed until any of the links is reopened, which typically leads to very long delays. Hence, in this sense, d is very important as a rerouting alternative to e. Links of this kind can be seen as the secondary-importance equivalents to cut links, and we may refer to them as *secondary cut links.*

Formally, let ΔT_{k+}^{ℓ} denote the total delay for all users that are redirected to k due to the closure of ℓ. By our earlier definition, these users amount to the flow f_{k+}^{ℓ}. Also, let $\Delta T_{k+}^{k\ell}$ be the total delay for the same users if both k and ℓ are closed. The delay-based secondary importance of k with respect to ℓ is defined as the difference

$$I_{2,\ell}^{\text{delay}}(k) = \Delta T_{k+}^{k\ell} - \Delta T_{k+}^{\ell} \tag{4}$$

It can be formally shown that the secondary delay-based importance of a link, with respect to a particular other link, will be large if either the redirected flow or the average difference in delay per user with and without the link itself being closed is large, and particularly if both are large.

3.4 Summarizing across other links

To obtain a single measure of the secondary importance of link k, we need to summarize $I_{2,\ell}^{\text{flow}}(k)$ and $I_{2,\ell}^{\text{delay}}(k)$, respectively, across all other links ℓ. One way is to consider the maximum values

$$\max_{\ell} I_{2,\ell}^{\text{flow}}(k) \quad \text{and} \quad \max_{\ell} I_{2,\ell}^{\text{delay}}(k). \tag{5}$$

This approach can be seen as a form of worst-case analysis. For the flow-based measure, this is useful if one wants to analyze whether k has the capacity to accommodate the largest flow that could potentially be added to the link. However, the maximum-value approach does not capture the fact that k may be an important alternative for several links. To capture this aspect, the most straightforward approach is to instead calculate the straight sums

$$\sum_{\ell} I_{2,\ell}^{\text{flow}}(k) \quad \text{and} \quad \sum_{\ell} I_{2,\ell}^{\text{delay}}(k). \tag{6}$$

In this way we include not only the worst-case link but also any other links for which the studied link serves as alternative. An implicit assumption here is that it is equally important to be an alternative to any other link. A more general approach is to use a weighted sum, i.e.,

$$I_{2}^{\text{flow}}(k) = \sum_{\ell} w_{\ell} I_{2,\ell}^{\text{flow}}(k) \tag{7}$$

and

$$I_{2}^{\text{delay}}(k) = \sum_{\ell} w_{\ell} I_{2,\ell}^{\text{delay}}(k), \tag{8}$$

where w_ℓ is a weight expressing the relative significance of being an alternative to link ℓ. Note that the maximum-value approach (5) is a special case of (7)–(8) with $w_\ell = 1$ if ℓ is the worst-case link and $w_\ell = 0$ otherwise. The straight-sum approach (6) is a special case with $w_\ell = 1$ for all links.

In the case study in this paper, we will mainly study a third summarizing approach in which we base the weight w_ℓ on the length of the link, denoted L_ℓ. One reason for this is that we use link length as a crude estimator for the relative probability that the link will be disrupted by an external event. All else being equal, it is thus considered more important to be a rerouting alternative for a link that is more likely to be closed. Another reason for weighting by link lengths is that it reduces the sensitivity of the analysis to how links and nodes are defined in the network representation of the real road transport system.

To make the importance measure easier to compare between different road networks of different sizes, we normalize the weights so that they sum to the average link length, $\bar{L} = \sum_\ell L_\ell / M$, where M is the number of links in the network. Thus, $w_\ell = L_\ell / \bar{L}$.

4 Case study

4.1 Specifications and data

We have calculated all importance measures presented in this paper for every link in a selected region of the Swedish national road network. The main study area consists of 18 municipalities in northern Sweden. This particular study area was selected since it was deemed to be of particular interest by representatives from the Swedish Road Administration.

The road network and the concentration of travel demand in and around the study area are shown in Figure 2. The population in the area is largely concentrated to the east coast, particularly to a few major towns, as well as one large inland town. The E4 European highway following the east coast is the largest road in the area. A few major roads start on the east coast and run inwards in the north-west direction. There is also a large inland road, the E45, that stretches roughly parallel to the E4 (see further [4]).

When considering the secondary importance, we have calculated the measures based on the maximum value (5), the unweighted sum (6) and the weighted sum (7)–(8) across all other links as defined in Section 3.2. Throughout the case study, we have assumed that the duration of the closures is 12 hours. For link pairs constituting two-way road segments, both directions are closed simultaneously, and results are presented for the link pair rather than for each direction separately. In order to reduce border effects when calculating the secondary importance measures, we have considered closures of links in twelve adjacent municipalities in addition to the study area itself.

The network and travel demand data (including both car and truck trips) were obtained from the Swedish national travel demand model system SAMPERS (Beser and Algers, 2001). For more information about this source of data,

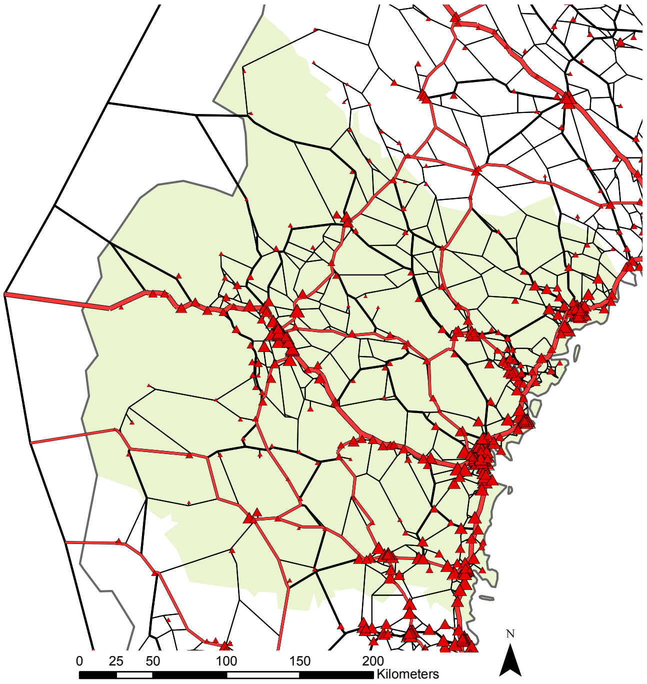

Fig. 2. The road network representation used in the case study including origin/destination nodes (triangles). The thickness of a link indicates road type (from highways down to minor roads), the size of a node indicates amount of outbound travel demand.

see [4, 2, 3]. Since we limit the analysis to northern Sweden, we used a network model that is coarser in the southern parts of the country to reduce the computation time. The road network representation as a whole consists of 8,529 nodes (including 1,399 OD nodes) and 23,878 directed links. The actual study area contains 5,888 directed links.

4.2 Delay calculations

To calculate the delays caused by link closures, we use the simple model described in detail in [2] and used in [3]. Essentially, we assume that link travel times are independent of link flows and that all travellers between origin i and destination j use the shortest available route (assumed to be unique) from i to j. We also assume that the travel demand per unit time between i and j is constant during the closure. The delay caused by a closure of any link k is then the difference in travel times on the shortest route from i to j with and without k being closed. However, if it is more worthwhile to postpone the trip until the links are reopened, travellers will do so. This adjustment of the model becomes effective in places where the road network is very sparse, so that it is unrealistic to assume

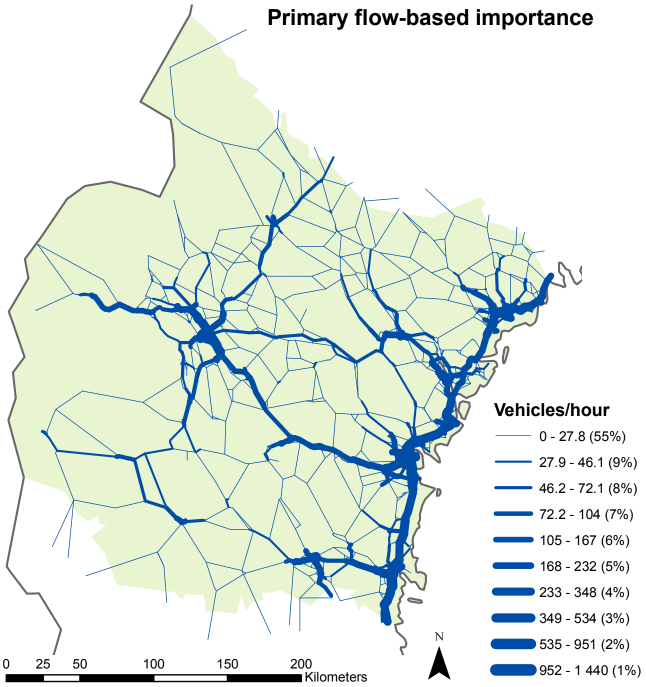

Fig. 3. Primary flow-based importance (1): normal link flow (vehicles/hour). Percentages indicate share of links in each category.

that all people embark on long detours rather than wait for the closure to be lifted. If the road network is dense, the adjustment becomes insignificant. If there are no available routes from i to j during the closure, we calculate the delay as the time from the intended departure time to when the link is reopened.

5 Results

5.1 Primary importance

Figure 3 shows the flow-based primary importance (1) of every link in the study area. The E4 European highway following the east coast can easily be identified in the figure by the large flows on its links. In the major towns, local roads have large flows and, to a lesser extent, so have the major roads connecting the population centres. Taken together, these roads can be said to form the backbone of the regional road network.

Figure 4 shows the primary delay-based importance (2) of every link, i.e., the total delay incurred when the link itself is closed. We see that many links with large flows, such as parts of the E4, are important in this sense as well. However, many less utilized links are also considered important due to the long detours

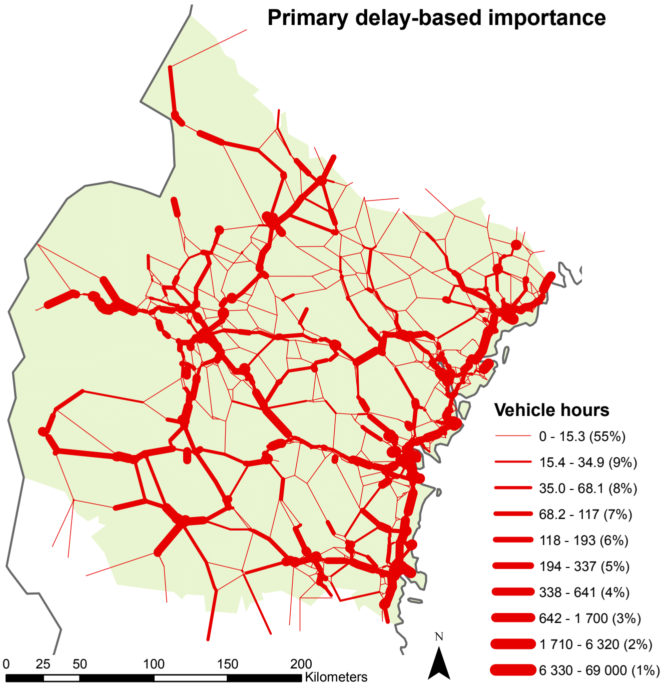

Fig. 4. Primary delay-based importance (2): total delay due to closure of the link itself (vehicle hours), 12 hour closure duration. Percentages indicate share of links in each category.

that have to be taken by those who use them. In particular, there a number of cut links scattered in the network. Since users of these links have to wait until the links are reopened, they are very important in the delay-based sense.

5.2 Secondary importance

Figure 5 shows the flow-based measure (7) calculated for every link in the study area using link lengths as weights. It appears that most links in the E4 are considerably less secondary important, in relative terms, than primary important (compare Figure 3). In contrast, many links running next to the E4 are considerably more secondary important. It is clear that these links, which normally have modest flows, serve as rerouting alternatives for the many travellers normally using the highway. That such links would be very secondary important is in line with our general hypothesis prior to the study.

A less intuitive results is that many links in the sparse southern inland part of the network are highly secondary important. To investigate the reason for this, we have also calculated the secondary link importance using the maximum-value and straight-sum summarizing methods (5) and (6). The analysis shows that these links are not very important when these approaches are used; instead, the

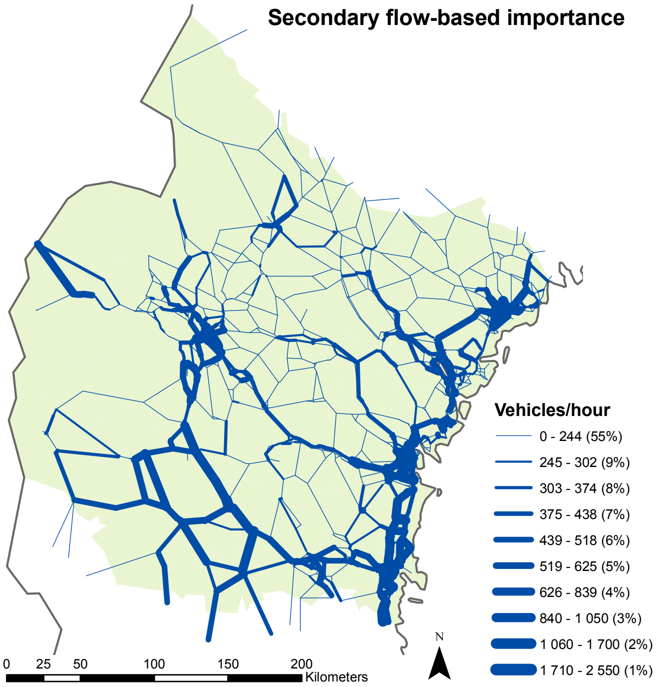

Fig. 5. Secondary flow-based importance (7): weighted sum of redirected traffic flows due to closure of another link (vehicles/hour), 12 hour closure duration. Percentages indicate share of links in each category.

links parallel to the E4 and links in and around the largest towns completely dominate. We can thus conclude that it is mainly the fact that many links in the southern inland are long, and hence assumed to be more likely to be subject to disruptions, that makes some links there important as alternatives.

Figure 6 shows the delay-based measure (8) calculated for every link, again using link lengths as weights. Compared to the flow-based measure in Figure 5, importance has shifted even more towards links with low normal traffic flows, in particular to links with long detours to the next best alternative route. In fact, many of these links are secondary cut links, i.e., links that are part of the only alternative route for at least one other link. As can be seen from Figure 2, many of these links are actually segments of roads that have been split into two or more links by demand nodes connecting to the road network. Hence, in a sense, these roads constitute their own alternatives.

It should be noted, however, that delay-based secondary importance, more so than the flow-based measure, is relatively sensitive to where the demand nodes connect to the road network in the network model. Hence, the results should be handled with some care.

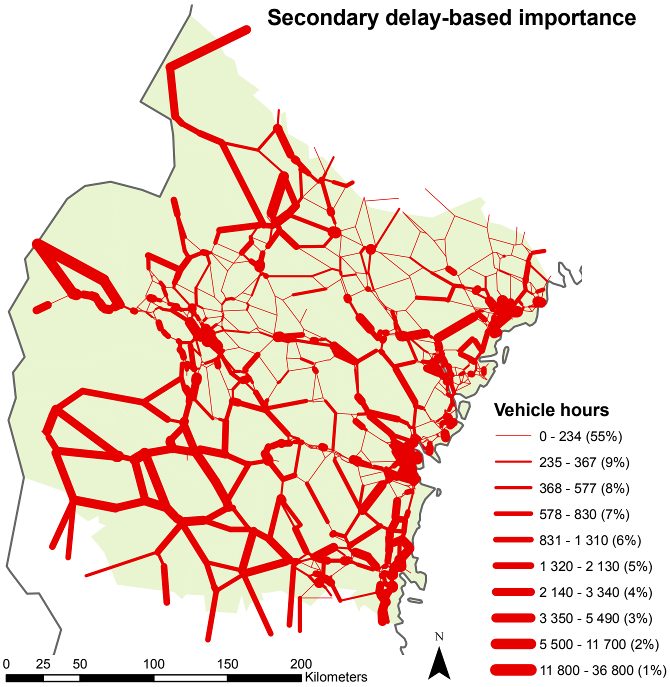

Fig. 6. Secondary delay-based importance (8): weighted sum of differences in total delay, with and without the link itself closed, for redirected traffic due to closure of another link (vehicles/hour). Percentages indicate share of links in each category.

6 Discussion

We argue in this paper that links that constitute valuable backup alternatives when other links fail should be considered important even if the impacts when they themselves fail are small. To quantify this notion of importance, which we call secondary importance, we have introduced two measures based on traffic flows and travel delays, respectively. These measures provide quantitative decision support in the planning of investments, maintenance and operations. They are also useful for setting up pre-emptive rerouting plans in order to relieve links where the probabilities or impacts of further failures would be large.

Which measure we should use in a given situation depends, we believe, somewhat on the circumstances. If we think that the failure of the other link is an isolated event and that there is no risk for further failures, then the flow-based measure should be sufficient. If, however, we think that there is a possibility that some link in the rerouting alternative will also be disrupted (either due to a common cause or due to a ripple effect of the first failure), then the delay-based measure may be appropriate.

We have restricted the importance measures here to failures of at most two links at the same time. It is straightforward to extend the analysis to joint failures of even more links. However, the larger number of links we consider, the more combinations of link failures do we need to calculate new routes for. Also, the probabilities that the links will be simultaneously disrupted should decrease.

A remaining issue is how much relative significance one should attribute to the different aspects of importance when prioritizing among projects. This is difficult to answer in exact terms, and may be a matter for political discussion. There are also other aspects of importance that we have not covered here, such as equity considerations [2]. However, we realize that the secondary importance measures capture, as the name suggests, secondary properties of the links, which may not warrant as high a priority as the performance of the network under normal conditions. Still, these properties should not be neglected as they certainly become critical in emergency situations.

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